The Design of Beam Pickup and Kickers

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Abstract

The behavior of beam pickup and kickers subjected to the electromagnetic fields of relativistic charged particles is examined. The concept of image currents is explained. The frequency domain response of a simple stripline pickup is derived and the behavior of kickers is explained through Lorentz reciprocity.

INTRODUCTION

Electromagnetic beam position pickups are an important component in the instrumentation of particle accelerators. Also, pickups and kickers are important to the design of beam feedback systems such as dampers and stochastic cooling systems. Pickups are usually designed to measure a particle's transverse or longitudinal position in a beam pipe by intercepting a portion of the electric and magnetic field that results from a moving charged particle. Kickers are basically pickups used in reverse. The electromagnetic field in a kicker is used to change a particle's transverse or longitudinal momentum.

The design of pickups is somewhat forgiving. Just about anything that is stuck in a beam pipe will intercept some portion of a beam's electromagnetic field. This accounts for the wide variety of pickups and kickers found in particle accelerators. However, the careful consideration of pickup and kicker parameters such as bandwidth, sensitivity, and impedance are important to the optimization of the design of beam position or damper systems. This paper will try to illustrate some of the fundamental principles in beam pickup and kicker designs. To keep the discussion clear and simple, this paper will for the most part consider the conventional stripline pickup and kicker.

IMAGE CURRENT

For most cases, the beam energy is much greater than the energy that is siphoned away in the pickup signal. That is, the pickup will not significantly decelerate the beam. With this in mind we can treat the beam as a current source. However, the beam charge will not directly flow through a pickup (unless the beam hits the wall of the beam pipe). The pickup will intercept some portion of the electromagnetic fields that accompany the beam.

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Any charged particle will exhibit an electromagnetic field. When the particle is at rest the field is electric and points radially outward in all directions as shown in Fig. 1a. As a particle moves it has both electric and magnetic fields. The way to calculate these fields is to transform the electric field of a particle in its own rest frame to the lab reference frame by using the relativistic electromagnetic tensor. Fortunately we do not need to go into this kind of detail in this paper. A qualitative view of a moving particle's field is shown in Fig. 1b. As a particle's velocity increases, the amount of electric field pointing in the direction of motion decreases and the azimuthal magnetic increases. The resulting field pattern starts looking like a flattened pancake whose width is inversely proportional to the particle energy. As the particle velocity approaches the speed of light or as the kinetic energy of the beam becomes much greater then the particle's rest energy, the direction of the electric and magnetic fields becomes transverse to the direction of motion(TEM).(1) To keep things simple, this paper will restrict its attention to high energy particle beams.

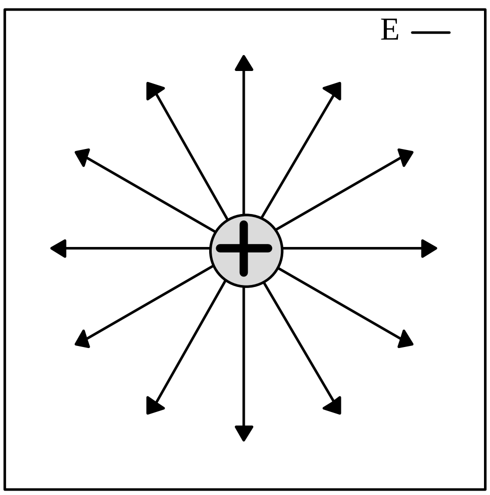

Figure 1a. Electric field of a charged particle at rest.

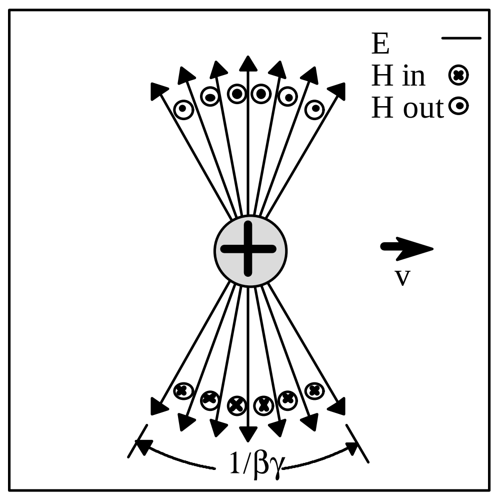

Figure 1b. Electromagnetic fields of a relativistic charged particle. (γ>>1)

When a charged particle is formed at a source, a particle of equal an opposite charge is also usually created. For example, a proton is extracted from a hydrogen atom by stripping the electron from the atom. This electron is still attracted to the proton and would like to follow the proton on its journey. As the proton is injected into the beam pipe (which we will assume is a metal for simplicity), the electron would like to "hop" onto the metal beam pipe. A metal can be considered as a sea of electrons that float around a lattice of positively charged ions. The velocity of these electrons is much smaller than the speed of light so that the electron that was injected onto the metal could not possibly keep up with the faster moving proton. (Note that due the nature of quantum mechanics one cannot really keep track of a particular electron in this "sea" of electrons.) The energy of the disturbance caused by the first electron "jumping" onto the metal is transmitted from electron to electron by means of an electromagnetic wave that follows the moving proton. This disturbance can be thought of as an image current flowing along the surface of the beam pipe that follows the proton. The magnitude of the image current is equal to the beam current but has the opposite sign.

The limiting case of a TEM pulse for a high energy particle beam electromagnetic field allows one to use Ampere's law to solve for the distribution of image current flowing on the walls of the beam pipe. Ampere's law with displacement current is:

$$\oint \vec{H} \bullet d\vec{l} = I_{beam} + \varepsilon \frac{\partial}{\partial t} \iint_{surface} \vec{E} \bullet \bar{z} \cdot dA \qquad (1)$$

where z is a unit vector that runs parallel to the beam and dl is a vector that is in a plane normal to the beam direction as shown in Fig 2. Since the electric field of the TEM wave is perpendicular to the beam direction, the displacement current term on the right side of Eq. 1 is zero. The transverse magnetic field can be found from the solution of the static two dimensional version of Ampere's law

$$\vec{\nabla} \times \vec{H} = \vec{J}_{beam} \qquad (2)$$

The transverse electric field is equal to the magnetic field multiplied by the wave impedance of free space:

$$\vec{E} = -\sqrt{\frac{\mu_o}{\varepsilon_o}} \bar{z} \times \vec{H} \qquad (3)$$

Since there are no electromagnetic fields in the metal of the beam pipe, an image current density must flow on the surface of the beam pipe to cancel out the magnetic field tangent to the metal surface.

$$\vec{J}_{image} = \bar{n} \times \vec{H} \qquad (4)$$

where J_S has the units of Amperes/meter and **n** is a unit vector that is normal to the beam pipe surface.(2)

Commercial computer programs that can solve the static two dimensional Ampere's equation are readily available. However, we will discuss two solutions in which the analytical solution is well known. For a cylindrical beam pipe with the beam current passing through the center of the pipe:

$$J_{image} = -\frac{I_{beam}}{2\pi b} \qquad (5)$$

If a pencil thin beam passes off center as shown in Fig. 3 (3):

$$J_{image}(\phi) = -\frac{I_{beam}}{2\pi b}\left[\frac{b^2 - r^2}{b^2 + r^2 - 2br \cdot \cos(\phi - \theta)}\right] \qquad (6)$$

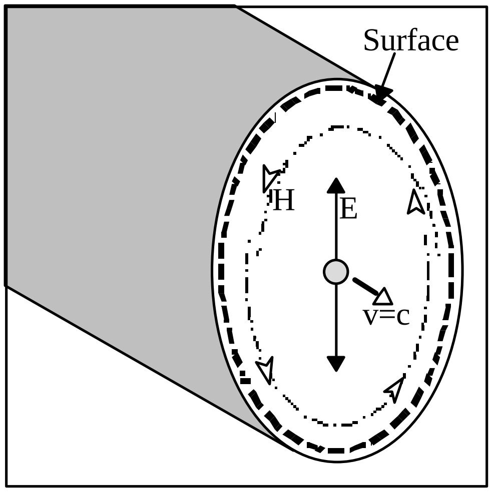

Figure 2. Beam pipe configuration for application of Faraday's Law.

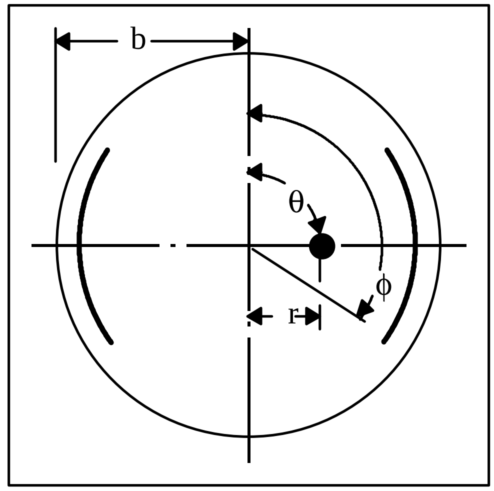

Figure 3. Beam going off center through the beam pipe. The dark lines on the sides are the pickup electrodes.

The image current density as a function of angular position for an offset pencil thin beam is shown in Fig. 4. If two electrodes are fashioned to intercept only a fraction of the image current as shown in Fig. 3, the electrode closer to the beam will intercept more image current compared to the further electrode. The difference in intercepted image currents between the two electrodes will be proportional to the beam position for small beam displacements. However, for large beam displacements, the difference signal will deviate from being linear with beam position as shown in Fig. 5. This situation can be corrected by fashioning the width as a function of electrode length.(3) We will define the sensitivity of the electrode as equal to the ratio of image current captured to the total image current.

$$s = \frac{\int_{width} J_s dw}{I_{beam}} \qquad (7)$$

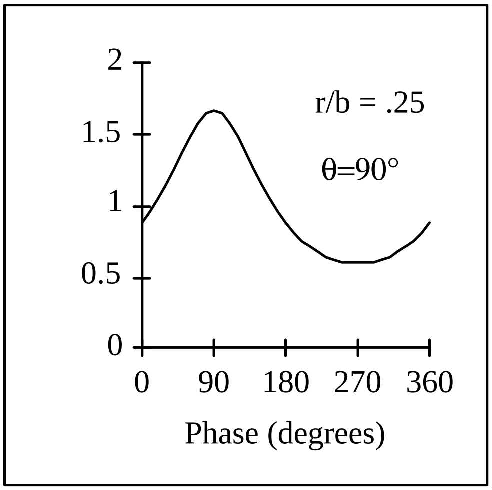

Figure 4. Image current density versus angular position for the electrode configuration shown in Fig. 3.

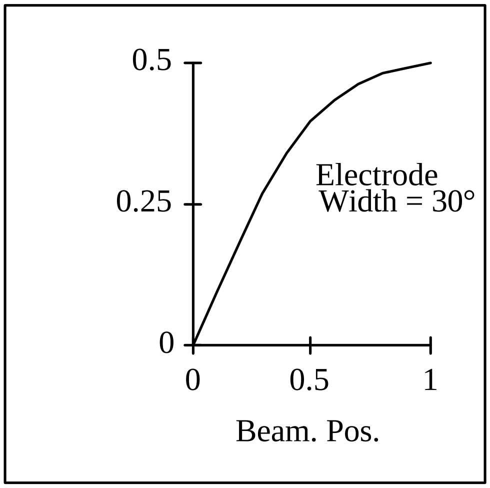

Figure 5. Difference in intercepted image current between the two electrodes shown in Fig. 3. versus actual beam displacement.

TIME DOMAIN RESPONSE

The following discussion will describe the time domain response of an extremely simple beam pickup. We will describe the electromagnetic fields in terms of integral quantities such as voltage and current.

Consider a moving charged particle in the center of a circular beam pipe traveling at a speed very close to the velocity of light as shown in Fig. 6. The observer is located at z=0. Assume that the particle will cross z=0 at time t=0. Because we are assuming that the particle's energy is much greater than its rest mass, the width of the pancake pattern of the EM fields as shown in Fig. 1b will shrink to infinitesimally narrow. The electrical current viewed by the observer can be approximated as:

$$i(t) = q \cdot \delta(t) \tag{8}$$

where δ is the Dirac delta function. The Dirac delta function $\delta(t)$ is zero everywhere outside t=0, infinite at t=0., and:

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \tag{9}$$

Because the δ function as it is written in Eq. 9, has units of 1/time, the current described by Eq. 8 has the correct units of charge/time. The image current which flows along the inner wall of the beam pipe is equal in magnitude but opposite in sign to the particle current. Also by symmetry, the current density is uniform around the entire circumference of the beam pipe.

Imagine that the beam pipe is sawed in half at z = 0. The gap between the two halves of the beam pipe looks like a capacitor to the image current(for infinitely long beam pipes this capacitance will be infinite). The only means for the image current to pass through the gap is by means of a displacement current which is analogous to an AC current flowing through a capacitor. If one could measure the voltage drop across the gap without disturbing the uniformity of the image current density, the voltage drop across the gap would be zero before t=0 and for t>0:

$$v(t) = -\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = -\frac{q}{C} \tag{10}$$

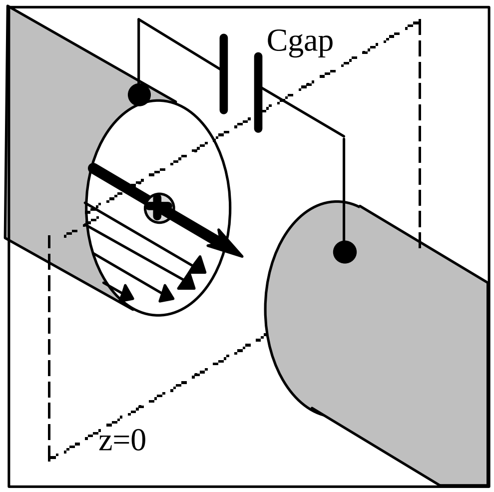

Figure 6. Beam pipe with capacitive gap cut at center.

THE FREQUENCY DOMAIN RESPONSE

The result of Eq. 10 is a simple result that could have been easily derived without going through the steps of Eqs. 8-10 but; the geometry of most pickups is not as simple as the configuration shown in Fig. 6. In general the voltage at the output of a pickup due to a single point charge is a function of time:

$$v(t) = q \cdot z(t) \tag{11}$$

where the units of z(t) is Ohms/second. For another particle that crosses the observation point at $t = t_p$, the voltage due to that particle is:

$$v_p(t) = q \cdot z(t - t_p) \tag{12}$$

If the beam contains more than one particle, the voltage is the sum of all the individual voltages generated by each particle.

$$v(t) = q\sum_p N_p z(t - t_p) \tag{13}$$

where N_p is the number of particles that cross z=0 at $t=t_p$. Up to this point we have been considering particles on a one by one basis. For most beams, there will be a large number of particles. The number of particles in a small slice of time called dt_p is:

$$N_p = \frac{i(t_p)}{q} dt_p \tag{14}$$

As the length of the slice approaches zero, the sum in Eq. 13 can be rewritten as an integral:

$$v(t) = \int_{-\infty}^{t} z\left(t - t_p\right) \cdot i\left(t_p\right) dt_p \tag{15}$$

Equation 15 is the convolution of the impulse response of the pickup with the beam current. Equation 15 is an important result. It implies that to completely describe the pickup all we need to know is the impulse response of the pickup z(t) and the charge distribution of the beam.

Coupled with the fact that convolution integrals are in general not easy to do and the availability of RF spectrum analyzers and network analyzers, it is much easier to solve Eq 15. in the frequency domain.(4) The Fourier transform of the voltage is:

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt \tag{16}$$

Since $V(\omega)$ is the spectral density of v(t), the units of $V(\omega)$ is Volts/Hertz. Applying the Fourier transform to Eq 15., we get Ohms law for pickups:

$$V(\omega) = Z(\omega) \cdot I(\omega) \tag{17}$$

where $I(\omega)$ is the spectral density of i(t) and has units of Amperes/Hertz. $Z(\omega)$ has units of Ohms and is called the pickup impedance. The rest of the paper will concentrate an how to calculate or measure $Z(\omega)$.

BEAM SPECTRUM

Before a pickup design can begin, the frequency spectrum of the beam current should be known. This section will discuss the spectrum of two simple beam profiles. These profiles are the square pulse and the gaussian bunch as shown in Fig. 7.

For the square pulse:

Time Domain		Frequency Domain	
$i(t) = \frac{N_b q}{2\sigma}$	for $\lvert t \rvert < \sigma$		
$i(t) = 0$	for $\lvert t \rvert > \sigma$	$I(\omega) = N_b q \frac{\sin(\omega\sigma)}{\omega\sigma}$	(18)

where N_b is the number of particles in the bunch. For a gaussian pulse:

Time Domain $\qquad\qquad$ Frequency Domain

$$i(t) = \frac{1}{\sqrt{\pi}} \frac{N_b q}{\sigma} e^{-\left(t/\sigma\right)^2} \qquad\qquad I(\omega) = N_b q e^{-\left(\omega\sigma/2\right)^2} \tag{19}$$

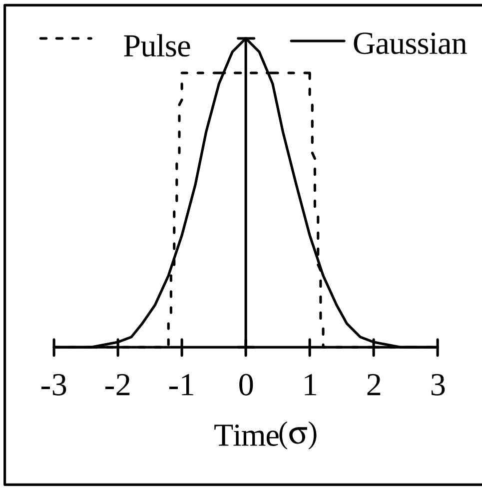

Figure 7. Time domain picture of a square pulse and gaussian waveform.

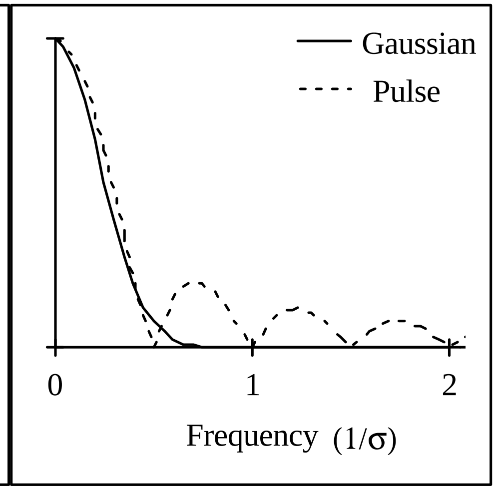

Figure 8. Fourier transform of a square pulse and a gaussian waveform.

The frequency response of these pulses is shown in Fig. 8. Note that the DC value for both cases is the total amount of charge in the bunch. The square wave pulse falls off much slower than the gaussian bunch at high frequencies. This is due to the sharp edges of the square pulse in the time domain. Although a perfect square pulse might be a simple approximation, one should also realize that the steep drop-off at high frequencies for a gaussian pulse is due to the fact that the tails of the gaussian in the time domain extend to infinity! In reality all beam pulses have a finite extent in time. Therefore the use of a gaussian bunch as a model for the bunch structure will not be accurate at high frequencies. In general any kind of edges in the distribution will cause the beam current spectral density to fall off as $1/\omega$ at high frequencies.

TRANSMISSION LINE THEORY

Because many pickup designs are based on transmission lines that couple to the beam, this section will give a brief over view of transmission line theory. A TEM transmission line consists of at least two conductors that run parallel two each but are separated by some small distance. Between the conductors there is stored magnetic energy which is proportional to the inductance of the conductors and there is electrical energy which is proportional to the capacitance between the two conductors. A small section of transmission line can be modeled as a series inductance with a shunt capacitance as shown in Fig. 9. The current going through the inductor is proportional to the voltage drop across the inductor:

$$v-(v+\Delta v)=L\cdot\Delta z\frac{\partial i}{\partial t}\quad\xrightarrow{\Delta z\to 0}\quad -\frac{\partial v}{\partial z}=L\frac{\partial i}{\partial t}\tag{20}$$

where L is the inductance per unit length. The current exiting the transmission line segment is the difference between the input current and the capacitance current (5):

$$i+\Delta i=i-C\cdot\Delta z\frac{\partial v}{\partial t}\qquad\xrightarrow{\Delta z\to 0}\quad -\frac{\partial i}{\partial z}=C\frac{\partial v}{\partial t}\tag{21}$$

where C is the capacitance per unit length. Equations 22 -23 govern how the voltage and the current "bootstraps" with each other along the transmission line. This "bootstrapping" can be written down as forward and reverse propagating waves:

$$\begin{aligned}v&=v^{+}\left(t-\frac{z}{vel}\right)+v^{-}\left(t+\frac{z}{vel}\right)\\ i&=\frac{v^{+}}{Z_o}\left(t-\frac{z}{vel}\right)-\frac{v^{-}}{Z_o}\left(t+\frac{z}{vel}\right)\end{aligned}\tag{22}$$

The (+) superscript indicates a wave in the +z direction and the (-) superscript indicates a wave traveling in the -z direction. The symbol **vel** is the phase velocity of the waves and is equal to:

$$vel=\frac{1}{\sqrt{LC}}\tag{23}$$

The symbol Z_o is called the characteristic impedance of the transmission line and is the ratio between the wave voltage and the wave current. It can be written in terms of the line inductance and capacitance as:

$$Z_o=\sqrt{\frac{L}{C}}\tag{24}$$

The first thing to note about Eq. 22 is that the form of the solution is very general. Thus, if a pulse is initiated on a transmission line its shape will not distort as it travels down the line. Also, the forward and reverse waves are independent modes. That is, if a forward wave is initiated on a transmission line it will remain a forward traveling wave as long as no discontinuity occurs on the transmission line.

However if a discontinuity occurs on the line, then both forward and reverse traveling waves must exist to support the boundary conditions governed by the discontinuity. For example, assume that a transmission line is terminated in some resistance R_t and a forward traveling wave has been initiated on the transmission line and is traveling towards the termination. At the termination, the ratio between the voltage and the current must be R_t. Therefore, the ratio between the forward and reverse waves can be found by dividing the top equation by the bottom equation in Eq. 22 and setting it equal to R_t:

$$\frac{v^-}{v^+} = \frac{R_t - Z_o}{R_t + Z_o} = \Gamma \tag{25}$$

This ratio is known as the reflection coefficient. After the forward wave hits the termination a reverse wave is set up to match the boundary condition and this wave heads back to the initial source of the waves. There are three limiting conditions. The first situation is when the termination is open or R_t= , the reflection coefficient is +1. A reverse wave of the same magnitude and polarity is sent back to the source. Next, if the termination is shorted or R_t=0, the reflection coefficient is -1. A reverse wave of the same magnitude but opposite polarity is launched back towards the source. Finally, if the termination R_t=Zo, the reflection coefficient is zero and no reverse wave is created. An observer looking into the load could not distinguish between a transmission line of infinite length and a line terminated with a resistor equal to its characteristic impedance.

In this paper we will be working mostly in the frequency domain so we will consider only sinusoidal waves. A forward traveling wave can be written as:

$$v^+(t,z) = V_o^+ \cdot \cos\left(\omega\left(t - \frac{z}{vel}\right)\right) = RE\left\{A \cdot e^{j\omega t}\right\} \tag{26}$$

Because of this simple time dependence, the operations of integration and differentiation become the simple operations of division and multiplication of jω:

$$\int dt \longrightarrow \frac{1}{j\omega} \qquad\qquad \frac{\partial}{\partial t} \longrightarrow j\omega \tag{27}$$

The quantity A is a complex number and can be thought of as a phasor where:

$$|A| = V_o^+ \qquad\qquad \arg(A) = -\omega \frac{z}{vel} \tag{28}$$

The negative sign for the phase is the result of choosing the positive sign for the time dependence jωt. The derivative of the phase with respect to frequency gives the time delay through the transmission line of length z:

$$-\frac{\partial}{\partial \omega} \arg(A) = \frac{z}{vel} = \tau_{delay} \tag{29}$$

Equation 28 holds for a simple transmission line. In general, the phasor "**A**" can be a complicated function of frequency. A vector network analyzer can be used to measure the variation of these wave phasors versus frequency. Most network analyzers can measure a circuit with one or two ports as shown in Fig 10. At each port, there is an ingoing wave "**A**" and an outgoing wave "**B**". The network analyzer measures the ratio of the outgoing waves to the ingoing waves:

$$\begin{aligned} B_1 &= S_{11}A_1 + S_{12}A_2 \\ B_2 &= S_{21}A_1 + S_{22}A_2 \end{aligned} \tag{30}$$

The matrix elements are called "S" parameters and are complex numbers. The diagonal matrix elements S_{11} and S_{22} are similar to the reflection coefficient described above. The off diagonal matrix elements S_{12} and S_{21} can be thought of transmission coefficients. (6)

THE STRIPLINE PICKUP

The stripline pickup shown in Fig. 11a is one of the most common types of beam pickup.(3) The metal electrode forms a transmission line along the direction in which the beam travels. The stripline is terminated at both ends with resistors equal to the characteristic impedance of the stripline. Note that both resistors do not have to reside inside the beam pipe. They can be brought outside by transmission lines with the same characteristic impedance and terminated outside the vacuum chamber.

As discussed in section on the frequency domain response, the pickup behavior can be characterized by its impulse response. As the impulse image charge follows the beam down the beam pipe, it will encounter the upstream edge of the stripline as shown in Fig. 11b. This edge looks similar to the gap discussed in section on the time domain response. However because the width of the stripline is only a fraction of the beam pipe circumference, the stripline will not intercept all of the image charge. The fraction that it does accept will be designated as **s**. This fraction of image charge that is intercepted by the loop will travel across the upstream gap as a displacement current. This displacement current will give rise to a voltage pulse on the upstream end of the stripline. The magnitude of the pulse is equal to the fraction of image current captured times the characteristic impedance of the stripline. Because the stripline is terminated at the upstream and downstream ends with the same characteristic impedance, the voltage pulse will split into two equal pulses with one of the pulses traveling to the upstream termination and the other pulse traveling to the downstream termination.

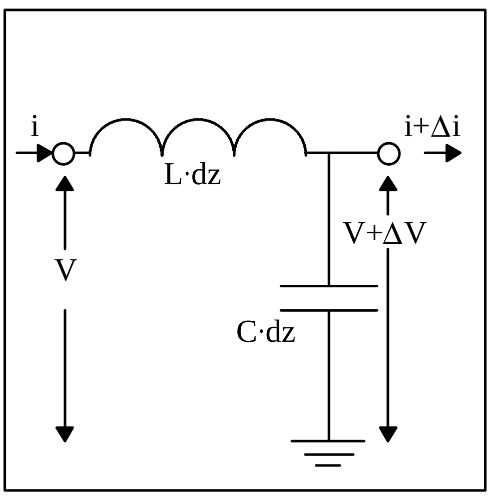

Figure 9. Electrical circuit representation of a small section of transmission.

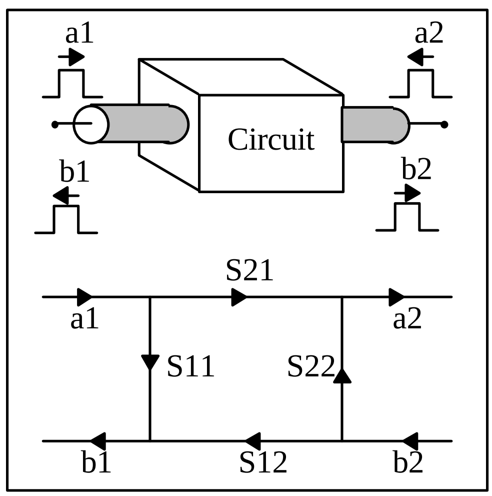

Figure 10. Definition of two port S parameters.

The fraction of image current traveling downstream will travel along the top of the stripline until it encounters the downstream gap. At the downstream gap another pulse is created. This pulse is equal to the upstream pulse but has the opposite polarity. This change in polarity is due to the fact that the electric field lines at the downstream gap start on the edge of the stripline and terminate on the ground plane. Whereas at the upstream gap, the electric field lines started at the ground plane and terminated on the upstream edge of the stripline.

The pulse at the downstream gap also splits into two pulses. However, if the velocity of the beam and the phase velocity of the stripline are equal (as in the case for high energy beams), the pulse that was created at the upstream gap and headed towards the downstream termination will arrive at the downstream gap at the same time the image current is inducing the downstream voltage pulse. Since the pulses heading towards the downstream termination have opposite polarity, these pulses will cancel each other and there will be no energy dissipated in the downstream termination! The pickup behaves very much like a microwave contra-directional coupler in that the direction of the energy flow in the pickup is in the opposite direction in which the beam is traveling. In practice some small fraction of energy will be dissipated because of small mismatches in velocity and discontinuities along the stripline.

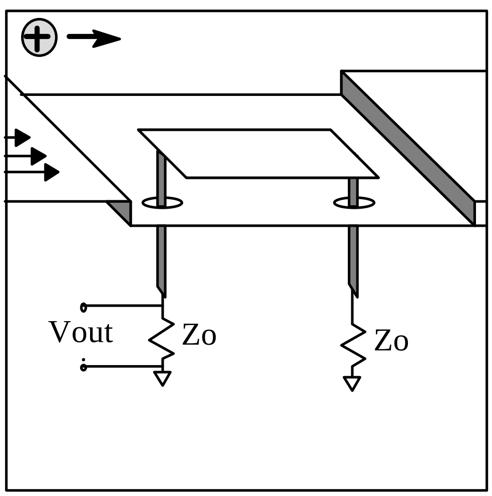

Figure 11a. A stripline pickup.

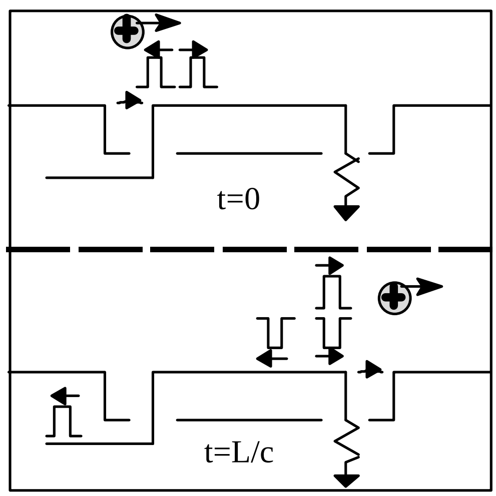

Figure 11b. Snapshot of induced voltage pulses on a stripline pickup.

The impulse response in the time domain which is sensed at the upstream termination consists of the first half pulse created at the upstream gap followed by a half pulse in the opposite polarity and delayed by twice the transit time of the stripline:

$$z(t) = \frac{s \cdot Z_o}{2}\left(\delta(t) - \delta\left(t - \frac{2 \cdot len}{c}\right)\right) \tag{31}$$

The Fourier transform of the impulse response is:

$$Z(\omega) = s \cdot Z_o \cdot e^{j\frac{\pi}{2}} \cdot e^{-j\frac{\omega \cdot len}{c}} \sin\left(\frac{\omega \cdot len}{c}\right) \tag{32}$$

which is plotted in Fig. 12. The delay of the response is equal to the length of the stripline. The phase intercept (phase when ω->0) is 90°. The magnitude of the response has a maximum at frequencies where the length is an odd multiple of quarter wavelengths:

$$f_{center} = \frac{1}{4}\frac{c}{len}(2n-1) \tag{33}$$

where n=1,2,3.... . The stripline pickup is usually designed to operate in the first lobe (n=1). The 3dB points of this lobe is:

$$f_{lower} = \frac{1}{2} f_{center} \qquad\qquad f_{upper} = 3 \cdot f_{lower} \tag{34}$$

which is greater than an octave of bandwidth! A simple frequency domain circuit model for this type of pickup is shown in Fig. 13.

Finally, at low frequency the response approaches:

$$Z(\omega) \xrightarrow{\omega\frac{len}{c} << 1} s \cdot j\omega L \cdot len \tag{35}$$

where L is the inductance per unit length of the stripline. Equation 35 tells us that the pickup actually looks like an <u>inductor</u> to the image current at low frequencies.

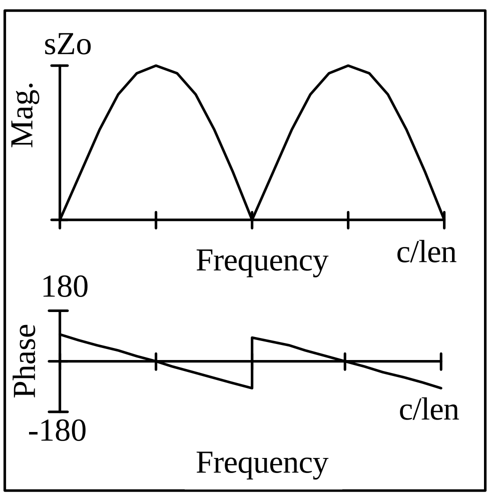

Figure 12. Frequency domain response of a stripline pickup.

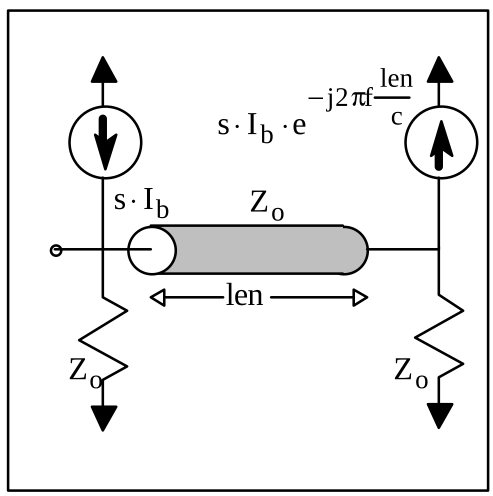

Figure 13. Equivalent electrical circuit of a stripline pickup.

HIGH FREQUENCY EFFECTS

As shown earlier, the pickup sensitivity increases as the width increases to intercept more image current. Also, the pickup sensitivity is proportional to the pickup impedance. However, the impedance of stripline decreases as the width increases. To combat this drop in impedance, the height of the stripline above the ground plane needs to be raised. The net result of increasing the pickup sensitivity and impedance is to make striplines that are wide and tall. In the analysis of the previous section, it was assumed that the image current distribution that was intercepted at the gaps formed a voltage pulse simultaneously and was represented as a delta function. That is, the current intercepted on the outside edges of the gap reached the termination at the same time as current that was intercepted in the middle of the gap. If the pickup width is an appreciable fraction of a wavelength at the frequency of interest, these assumptions are no longer valid.(7) An extremely simple model of a impulse response for a pickup that is very wide is shown in Fig. 14a. This model assumes that the image current at the outside edges of the gap take a longer time to reach the termination as compared to image current intercepted at the centers of the gap. The frequency response is given as:

$$Z(\omega) = s \cdot Z_o \cdot e^{j\frac{\pi}{2}} \cdot e^{-j\omega\left(\frac{len}{c}+\sigma\right)} \frac{\sin(\omega\sigma)}{\omega\sigma} \sin\left(\frac{\omega \cdot len}{c}\right) \qquad (36)$$

where σ is equal to the delay time between the center and the outside edges of the gaps. This response function is plotted in Fig. 14b. The response is equal to the frequency response of the square pulse described in Eq. 18 times the impulse response of Eq. 32. One should note that Eq. 38 collapses down to the original impulse response described in Eq. 36 when σ approaches zero. As shown in Fig. 14b, the spreading of the image charge on the gap has the effect of lowering the bandwidth of the response. So, as common to many different types of electrical engineering, there is a upper limit on the gain-bandwidth product of the performance of the pickup.

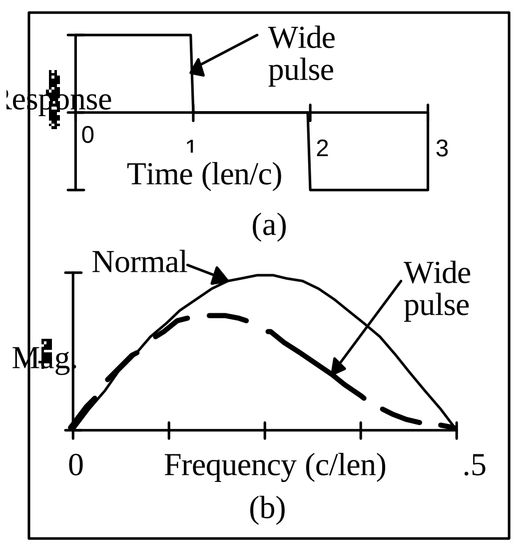

Figure 14. Simple time and frequency domain impulse responses of a pickup with transit time delay along gaps included.

For extremely high frequency pickups such as stochastic cooling arrays, the band limiting effect of the transit time delay due to electrode width is a serious limitation. Also, the appropriate transit time delay is difficult to calculate due to the three dimensional behavior of the stripline pickup. One should also note that we have completely avoided the discussion of parasitic inductances and capacitances due to end effects of the stripline electrode. However, some of these problems may be made more tractable by changing the three dimensional geometry of the stripline pickup to a planar, two dimensional, pickup as shown in Fig. 15 (8).

The gaps for this electrode can be modeled approximately by slotline transmission lines. Slotline transmission lines do not support TEM waves so that a unique characteristic impedance cannot be defined. However, a power impedance of the form:

$$Z_o \equiv \frac{\left| \int_{gap} \vec{E} \cdot d\vec{l} \right|^2}{2P} \tag{37}$$

where E is the electric field across the gap and P is the power transmitted along the slotline. A circuit model for this structure is shown in Fig. 16 where the weighted image current is separated by transmission lines. Note that if the transmission lines on the transverse gaps were omitted, this circuit model would reduce to the simple stripline model shown in Fig. 13. The overall effect of this model is to describe the electrode with an effective length.

$$\text{Length}_{\text{effective}} = \text{Length} + \text{Width} \tag{38}$$

Pickups with bandwidths up to 4 GHz have been fabricated with this geometry. At these high frequencies, the physical length shrinks almost to zero and most of the delay is made up in the width of the electrodes. As the length shrinks to zero, the electrode becomes the magnetic dual of a folded dipole antenna.

KICKERS AND RECIPROCITY

Up to this point there has been no discussion on the behavior of kickers. However, using the Lorentz reciprocity theorem, the behavior of kickers can be understood from the pickup behavior. The Lorentz reciprocity theorem is a useful and well known theorem for describing antenna systems. Lorentz reciprocity is derived by manipulating two solutions of Maxwell's equations. One solution describes the transmitter (kicker) and the other solution describes the receiver (pickup) (9).

Consider the receiver or pickup configuration in Fig. 17a. The beam which is represented as a current source density J_p induces an electric field at the pickup E_p. The current source also induces fields E_p,H_p throughout the beam pipe volume which is surrounded by a surface S. Figure 17b describes the transmitter or kicker configuration. The kicker is powered by a current source density J_k and produces a field E_k that acts on the beam. Also there are fields throughout the volume designated by E_k, H_k. By manipulating Maxwell's equations describing the two configurations, the identity:

$$\oiint_{\text{surface}} \left(\vec{E}_k \times \vec{H}_p - \vec{E}_p \times \vec{H}_k\right) \bullet d\vec{s} = \iiint_{\text{volume}} \left(\vec{E}_p \bullet \vec{J}_k - \vec{E}_k \bullet \vec{J}_p\right) dvol \qquad (39)$$

is derived. The contribution to the surface integral on the left hand side of Eq. 39 at the beam pipe walls is zero because the electric fields for both cases vanish. Also, the surfaces in which the beam enters and exits are far enough away from the electrode so that the fields are normal to the beam direction and this contribution to the surface integral vanishes. Equation 39 can be rewritten as:

$$\iint \vec{J}_k \left(\int \vec{E}_p \bullet d\vec{l} \right) \bullet d\vec{S} = \iint \vec{J}_p \left(\int \vec{E}_k \bullet d\vec{l} \right) \bullet d\vec{S} \qquad (40)$$

which can be re-written in integral form as:

$$I_k V_p = I_p V_k \qquad (41)$$

The pickup impulse response is the ratio of the voltage induced on the pickup to the beam current. The kicker impulse response is the ratio the longitudinal energy given to the beam to the current in the kicker. Equation 41 can be re-arranged

$$Z_p = Z_k \qquad (42)$$

Which states that to know the impulse response of the kicker all one has to do is measure (or derive) the impulse response of the pickup! However, it should be noted that due to the "dot" products in Eq. 40, this theorem holds only for kickers that give an accelerating kick in the direction of beam travel (i.e.. longitudinal kickers). Also as in the case of the stripline pickup above, a stripline kicker will be contra-directional. That is the voltage wave in the kicker starts at the downstream end of the electrode and travels to the upstream end.

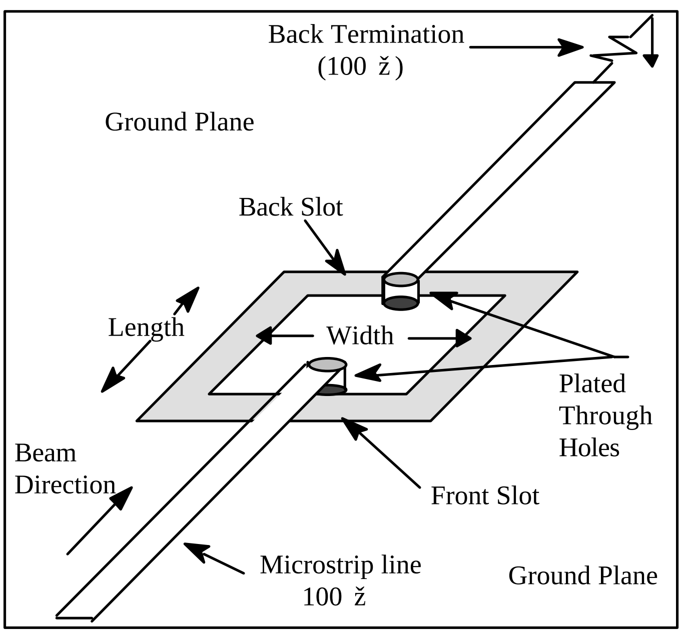

Figure 15. Simple geometric model of a planar loop

For transverse kickers another theorem using Maxwell's equations can be derived. However, we will use some simple arguments to derive the transverse kicker response for a stripline kicker.

A longitudinal stripline kicker is shown in Figure 18a. Both the top and bottom electrodes are wired in the sum mode. The pickup response and therefore the kicker response is zero at DC as discussed in Eq. 32. This can be simply seen by noting that at low frequencies there is no phase delay through the stripline so that the electric field along the beam direction at the upstream gap cancels the electric field at the downstream gap. There is no net acceleration. However, for a transverse kicker the electrodes are wired in the difference mode as shown in Fig. 18b so there is a net transverse electric field at DC. To account for this DC transverse field, Eq. 32 must be modified:

$$Z_{\perp}(\omega) = s \cdot Z_o \cdot e^{-j\frac{\omega \cdot len}{c}} \frac{\sin\left(\frac{\omega \cdot len}{c}\right)}{\left(\frac{\omega \cdot len}{c}\right)} \qquad (43)$$

This is formally derived by the Panofsky - Wenzel theorem which relates the transverse impedance to the longitudinal impedance with a factor of $1/j\omega$ (10). The response of a longitudinal and transverse kicker is shown in Fig. 19. Note that although the kicker has a DC response, the upper -3dB frequency is 40% lower than the upper -3dB frequency for the longitudinal kicker because of the $1/j\omega$ factor of the transverse field.

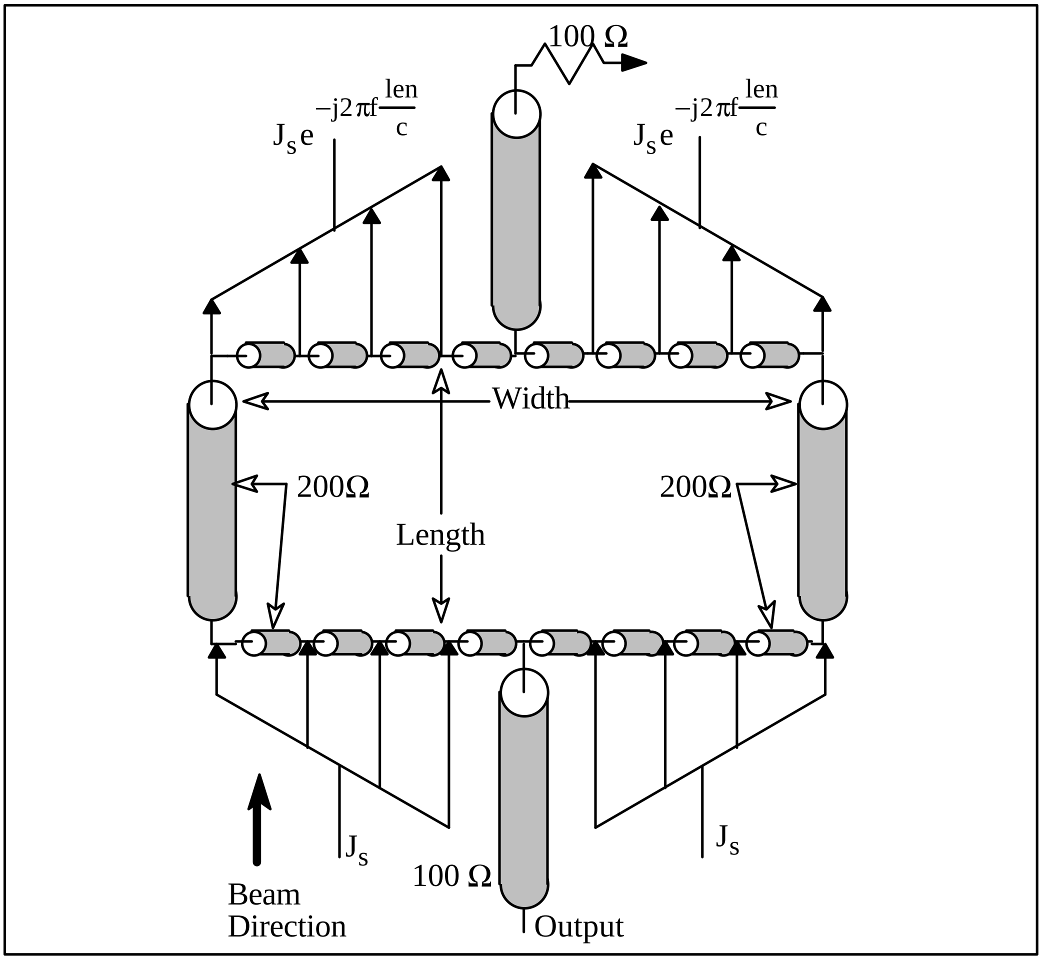

Figure 16. Equivalent electrical circuit for a planar loop

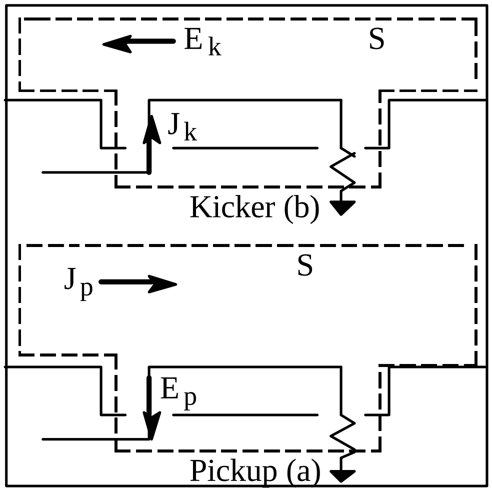

Figure 17. Lorentz reciprocity for a longitudinal pickup and kicker.

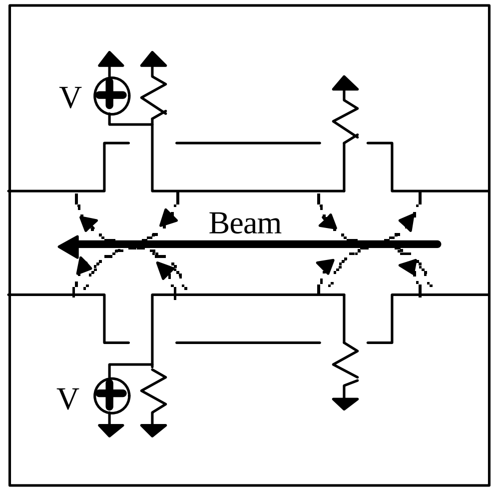

Figure 18a. Kicker wired in the sum mode (longitudinal).

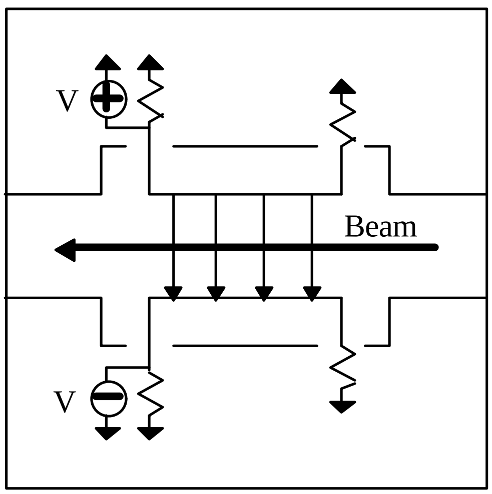

Figure 18b. Kicker wired in the difference mode (transverse).

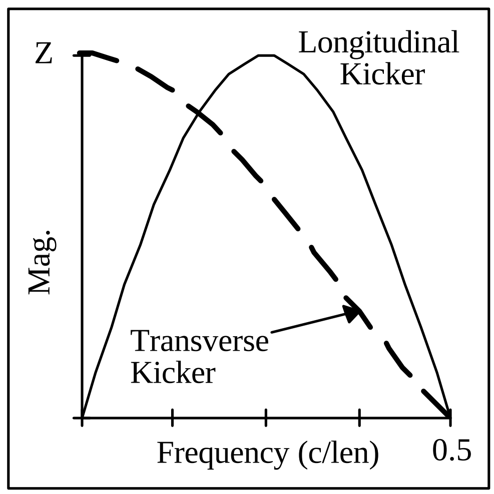

Figure 19. Response of a longitudinal and transverse kicker.

WIRE MEASUREMENTS

After an pickup or kicker electrode has been designed, it is often desirable to test the performance of the electrode before it is installed in the accelerator. The transverse electromagnetic (TEM) fields of the beam can be simulated by stretching a wire over the electrode in the direction of beam travel. The wire forms a TEM transmission line with the ground plane of the electrode with a characteristic impedance of Z_O which is a function of the wire diameter and its distance away from the ground plane. Downstream of the pickup, the wire is terminated with the ground plane with a resistor equal to Z_O. The electrical schematic of a wire measurement is shown in Fig. 20.

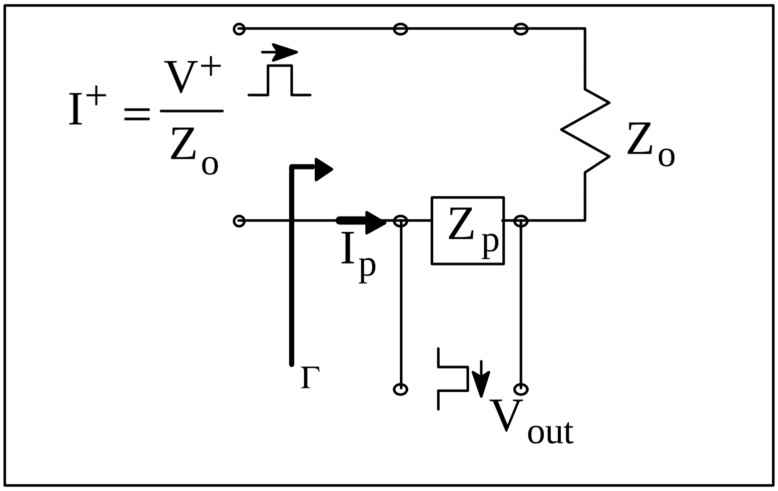

Figure 20. Electrical schematic of a wire measurement

The impedance seen by the forward traveling wave at the pickup is the pickup impedance plus the characteristic impedance of the transmission line. Because this net impedance is not equal to the characteristic impedance of the wire transmission line, a portion of the incident wave will be reflected at the pickup backwards towards the source. In the case of the TEM wave due to an actual charged particle beam, this reflected wave could not exist because a charged particle beam cannot support a TEM wave traveling in the reverse direction! The only waves that travel in the reverse direction for a particle beam are the higher order waveguide modes of the beam pipe. The frequencies of usual interest are usually below the cutoff of these higher order modes.

The effect of this reflected wave for the wire measurement can be determined by calculating the network analyzer transfer function S21.

$$S_{21} = \frac{V_{out}}{V^+} \tag{44}$$

The current flowing into the electrode for the wire model is:

$$I_p = I^+ \cdot (1 - \Gamma) \tag{45}$$

where:

$$\Gamma = \frac{(Z_p + Z_o) - Z_o}{(Z_p + Z_o) + Z_o} \tag{46}$$

The transfer response is:

$$\frac{V_{out}}{V^{+}} = S_{21} = \frac{1}{Z_o + \frac{1}{2} Z_p} Z_p \qquad (47)$$

which can be inverted to find the electrode impedance:

$$Z_p = \frac{S_{21}}{1 - \frac{1}{2} S_{21}} Z_o \qquad (48)$$

For simplicity, we have assumed that the pickup electrode presents a lumped impedance Z_p to the wire transmission line. In reality, this is a gross oversimplification because the pickup is often operated at frequencies in which the electrode is a quarter wavelength long. Therefore, the terms in the denominators of Eqs. 47 and 48 are not accurate.

However, the denominator terms of Eqs. 47 and 48 can be neglected if the characteristic impedance of the wire is made much larger than the impedance of the electrode. This is difficult to due in practice. The characteristic impedance of the wire transmission line only increases logarithmically as the diameter of the wire is decreased so characteristic impedances greater than 400Ω are difficult to obtain. Also the measuring ports of network analyzers have a characteristic impedance of 50Ω. In order to avoid reflections caused by an abrupt change in characteristic impedance between the network analyzer cable and the wire transmission line, a matching network must be built between the network analyzer and the wire transmission line. The bandwidth of the matching networks are usually inversely proportional to the difference in characteristic impedance between the network analyzer and the wire transmission line so that broadband measurements are difficult to make.

Another method to avoid the denominator terms in Eqs. 47 and 48 is to reduce the electromagnetic coupling (S_{21}) between the wire and the electrode. This is fairly easily done for the measurement of beam position electrodes. To measure the beam position, two electrodes are placed on opposite sides of the beam pipe and the difference between the two signals is taken as shown in Fig. 21a. Because of the odd symmetry of the measurement, the measurement will not be sensitive to any longitudinal component of the electric field at the center plane between the two electrodes. Therefore one of the electrodes can be replaced with a ground plane inserted between the two electrodes as shown in Fig. 21b. The ground plane in Fig. 22 preserves the odd symmetry of the measurement shown in Fig. 21a. The beam in Fig. 22 can be modeled with a wire over the ground plane as shown in Fig. 21c. The wire size and spacing is chosen to make a 50Ω transmission line. The 50Ω transmission line configuration of Fig. 21c avoids the mismatches between the network analyzer and the wire. If the wire is placed sufficiently close to the ground plane, the coupling will be reduced so that:

$$S_{21} << 1 \qquad (49)$$

The parameter of interest for a beam position pickup is the ratio of the impedance to the transverse displacement of the wire which is shown in Fig. 21c as **x**.

$$Z_p(\Omega / \text{mm}) = \frac{S_{21}}{x} Z_o \tag{50}$$

One important mistake that is often made with wire measurements is to forget to connect the ground of the network analyzer coaxial cable to the ground plane of the electrode. This will cause a serious disturbance to the image currents flowing on the electrode ground plane which will distort the measurement. This connection is emphasized by heavy lines in Fig. 21c.

TIME DOMAIN GATING

Even when there is a good ground connection between the network analyzer and the wire transmission line, there will be reflections due to the mismatch in geometry between the analyzer cables and the wire transmission line. These reflections will introduce some errors in the measurements. The response shown in Fig. 22a is typical of a measurement were there are some unwanted reflections. The reflections show up as "noise" or ringing added on top of the desired signal. The spacing in frequency between the notches of the ringing is inversely proportional to the distance between the electrode and the source of mismatch.

Most network analyzers have a time domain option built in. The impulse response in the time domain is obtained by performing the inverse Fourier transform on the frequency response of S_{21}. This response is shown in Fig. 22b. Besides the normal doublet response as described in Eq. 31, there is a small reflection of the response that comes at a much later in time. In the time domain option of certain network analyzers, a "gate" can be put around the desired signal as shown in Fig. 22b. Then, when the Fourier transform is applied to the gated signal the ringing is removed from the frequency response as shown in Fig. 22c.

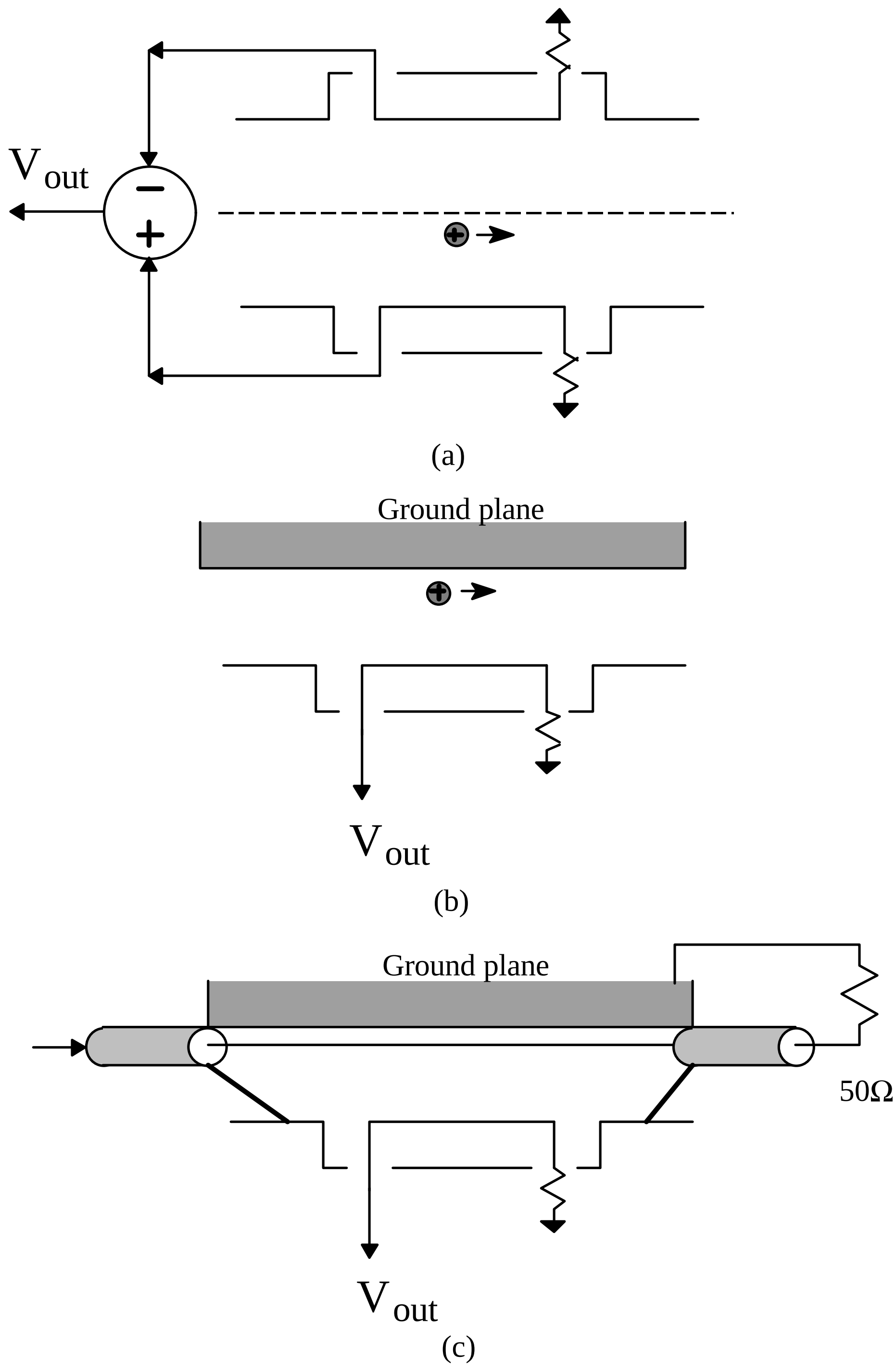

Figure 21. Low coupling difference mode wire measurement.

SUMMARY

A simple understanding of beam pickup and kickers can be obtained as considering a relativistic beam as a current source. The current source that impinges on the electrodes can be thought of as an image current flowing along the beam pipe walls or a TEM wave that flows through the beam pipe. The response of a pickup to any input signal can be calculated by using the impulse response of pickup. The difficult operation of convolution in the time domain turns out to be multiplication in the frequency domain. The high frequency behavior of pickups can often be explained in terms of transmission line concepts. The simple one dimensional model of a stripline electrode breaks down when the width of the stripline is on the order of a wavelength for the frequency of interest. Extremely high frequency electrodes can be made using two dimensional planar electrodes which account for transit time delays along the edges of the pickup. Using Lorentz reciprocity, the behavior of longitudinal kickers can be understood studying the pickup response. However, for transverse kickers, a division of $j\omega$ must be applied to the pickup response in order to get to true kicker response.

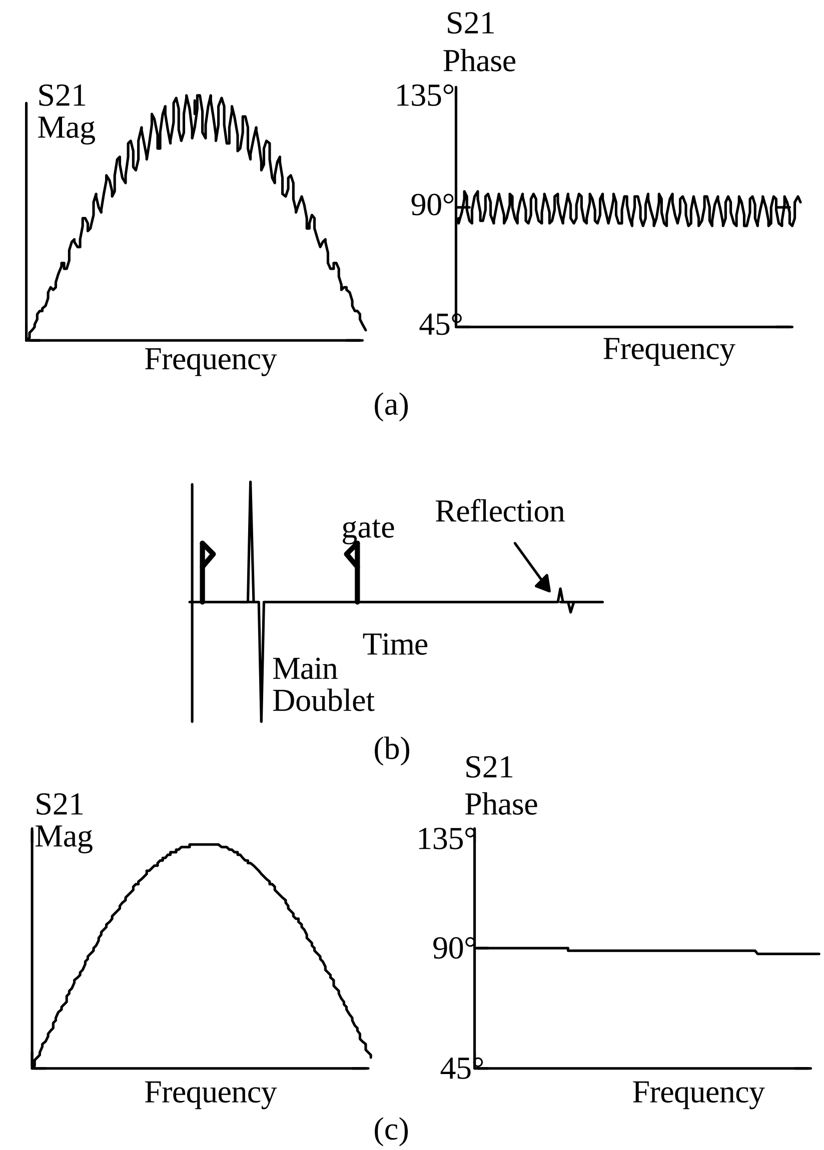

Figure 22 . Time domain gating.

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